Thomson Scattering

H.-J. Kunze

Institute for Experimental Physics V, Ruhr-University, 44780 Bochum, Germany

Conference, Université Pierre et Marie Curie, Paris, November 23rd, 2011
Injection of a laser beam into a plasma allows localized measurements.

Information due to scattering is from the gray volume in the Plasma, plasma radiation is collected along the line of sight.
Laser beams interact with electrons, ions, atoms, molecules.

Due to tunability and possible high power of lasers, interactions are numerous inclusive heating of plasma.

a) $\Rightarrow$ Modification of the primary transmitted beam
b) $\Rightarrow$ Scattering

a) Laser atomic absorption spectrosocpy (LAAS) with tunable diode lasers much applied in low density plasmas.
modification of the phase \( \Rightarrow \) interferometry 

plane of polarization \( \Rightarrow \) Faraday rotation 

\[ \int n_e ds \]

\[ \int n_e B ds \]

Scattering by atoms, ions, and molecules
- laser induced fluorescence (LIF)
- Rayleigh scattering
- Raman scattering
- Coherent anti-Stokes Raman scattering (CARS)

Scattering by electrons \( \cdots \) Thomson scattering
Since 1963 Thomson scattering by plasma electrons matured into one of the most powerful diagnostic methods for the determination of plasma parameters.

Latest development: employing x-rays at near solid state density plasmas scattering experiments on warm density matter using a FEL.

An extensive introduction and comprehensive review of the field has been published only recently.

D. H. Froula, S. H. Glenzer, N. C. Luhman Jr., J. Sheffield
*Plasma Scattering of Electromagnetic Radiation – Theory and Measurement Techniques*
Elsevier Inc. Amsterdam, 2011
Incoherent Thomson scattering

Principles
Theory is well understood:

Electromagnetic waves are focused into the plasma

⇒ charged particles oscillate and radiate like dipoles;

because of small mass \( m_e \ll M \) acceleration of the ions is much smaller and

essentially only electrons contribute

⇒ scattered radiation

Cross-section

\[
\sigma_{Th} = \frac{8}{3} \pi r_e^2 \approx \frac{2}{3} \times 10^{-24} \text{ cm}^2
\]

is extremely small
⇒ to have enough scattered photons one needs
  • long observation times (stationary plasmas) or
  • high intensity of incident radiation
⇒ scattered radiation should be clearly above plasma background radiation
⇒ lasers

At low densities all electrons scatter independently
no correlation between the electrons
scattered intensities simply add

\[
\frac{d\sigma}{d\Omega} = n_e \sigma_e = n_e r_e^2 \sin^2 \phi = n_e \frac{3}{8\pi} \sigma_{Th} \sin^2 \phi \propto n_e
\]
In the plasma electrons move fast ⇒ considerable Doppler shift

Doppler shift has to be considered twice

Incoming wave \((\omega_o, \vec{k}_o)\)

Scattered wave \((\omega_s, \vec{k}_s)\)

Scattering angle \(\angle(\vec{k}_s, \vec{k}_o) = \theta\)

\[
\omega = \omega_s - \omega_o = \vec{k} \cdot \vec{v}
\]

\[
\vec{k} = \vec{k}_s - \vec{k}_o
\]

Multiplication with \(\hbar\)

Equations correspond to conservation of energy and momentum

\(\vec{k}\) scattering vector
\[ \omega = \omega_s - \omega_o = \vec{k} \vec{v} = k \, v_k \]

Only the component $v_k$ in the direction of the scattering vector $\vec{k}$ is responsible for a Doppler shift!

hence

The Doppler broadened scattered radiation mirrors the one-dimensional velocity distribution function $f_k(v_k)$

\[ k \approx 2k_o \sin \frac{\theta}{2} \approx \frac{4\pi}{\lambda} \sin \frac{\theta}{2} \]

\[ \vec{k}_s \approx \vec{k}_o \]
\[ v_k = \frac{(\omega_s - \omega_o)}{k} \quad \Rightarrow \quad f_k(v_k) \quad \Rightarrow \quad f_k\left(\frac{\omega_s - \omega_o}{k}\right) \]

Fraction of electrons \( dn_e \) giving the same Doppler shift

\[
 dn_e = n_e \ f_k(v_k) \ dv_k = n_e \ \frac{1}{k} \ f_k\left(\frac{\omega_s - \omega_o}{k}\right)
\]

Differential scattering cross-section

\[
 \frac{d^2 \sigma}{d\Omega_s d\omega_s} = \frac{3}{8\pi} \ n_e \ \sigma_{Th} \ \sin^2 \varphi \ \frac{1}{k} \ f_k\left(\frac{\omega_s - \omega_o}{k}\right)
\]

\( n_e \) and \( f_k \) are measured
Maxwellian velocity distribution function
scattered profile is of \textit{Gaussian} shape

FWHM

\[
\Delta \lambda_{1/2} = 4 \lambda_0 \sin \frac{\theta}{2} \left( \frac{2k_B T_e}{m_e c^2} \ln 2 \right)
\]

Example: \( \lambda_0 = 694.3 \text{ nm (ruby laser)} \) and \( \theta = 90^\circ \)

- \( k_B T_e = 100 \text{ eV} \) \( \Delta \lambda_{1/2} = 32.4 \text{ nm} \)
- \( k_B T_e = 1 \text{ eV} \) \( \Delta \lambda_{1/2} = 3.24 \text{ nm} \)

Large widths! Low resolution instruments suffice

High temperatures? Relativistic effects!
At higher temperatures → relativistic treatment

**SCATTERED SPECTRUM**

ARL. UNITS

- 1 KEV
- 3 KEV
- 10 KEV
- 20 KEV

CALCULATED SCATTERING SPECTRA AT $\theta = 90^\circ$ FOR VARIOUS TEMPERATURES.
INCIDENT WAVELENGTH $\lambda_0 = 694.3$ nm.

FIG. 5: Scattering spectrum obtained from one discharge for a high-density plasma ($\sim 10^{16}$ cm$^{-3}$). The shift parameter is obtained from a least squares fit to a Gaussian. The first-order relativistic correction predicts a $\sim 9.4$ Å shift. The temperature is determined from a fit to the relativistically corrected Gaussian.

K. C. Maffei and H. R. Giem 3030
Problems:

At low densities: few scattered photons $\rightarrow$ noise problem

photon counting detection system

Stray light: elaborate baffle systems at entrance and exit port of laser beam

triple spectrograph eliminates center wavelength

Absolute calibration: on the entrance side 100 MW and up scattered radiation $\mu$W and lower

Trick: Calibration of the complete system by Rayleigh or Raman scattering in gases, cross-sections are well known

Heating: by of absorption of the laser beam
Scattering on large fusion devices

Salzmann and Hirsch proposed a backscattering scheme employing a sub-nanosecond laser pulse and time-of-flight analysis with high-speed detectors

LIDAR: Light Detection And Ranging

First implemented on JET, laser duration was 300 ps (9 cm long!)
Collective scattering

In the general case of scattering the electric fields of all scattered waves have to be added. The differential scattering cross-section becomes

\[
\frac{d^2\sigma}{d\Omega d\omega_s} = r_e^2 \sin^2 \varphi \ n_e \ S(k, \omega)
\]

The first two factors are the well known scattering cross-section for a single free electron

\( r_e \) is the classical electron radius

\( S(k, \omega) \) is known as the dynamic form factor or spectral density function, it contains the properties of the whole ensemble of electrons

Now correlations between the electrons and between electrons and ions show up in \( S(k, \omega) \)
An instructive and useful approximation of $S(k, \omega)$ was introduced by Salpeter in 1960.

A measure of the influence of correlations is $1/k$, i.e. the distance over which the particles are sampled, in relation to the Debye length

$$\frac{1}{k} \ll \lambda_D \Rightarrow k\lambda_D \gg 1 \Rightarrow \alpha = \frac{1}{k\lambda_D} \ll 1 \quad \text{no correlations}$$

$$\frac{1}{k} > \lambda_D \Rightarrow k\lambda_D < 1 \Rightarrow \alpha = \frac{1}{k\lambda_D} > 1 \quad \text{correlations}$$

$\alpha$ is called the scattering parameter.
Change of dynamic form factor $S(\alpha, x)$ with increasing scattering parameter $\alpha$

A narrow central part of high magnitude → ion feature
Its width is determined essentially by the mean ion velocity
Two sidebands shifted by about the electron plasma frequency $\omega_{pe}$

In the Salpeter approximation

$$S(k,\omega) = S_e(k,\omega) + S_i(k,\omega)$$

Electron feature: reflects electron plasma waves

Bohm-Gross dispersion relation

$\longrightarrow \ \omega_{pe} \ \longrightarrow \ n_e$

Around $\alpha \sim 1$

shape of electron feature changes very much and
shape alone gives $n_e$ and $T_e$, no absolute calibration necessary!
Problem at large $\alpha$:  
\[ S_e(k) = S_e(k, \omega)d\omega = \frac{1}{1 + \alpha^2} \text{ small} \]

hence In many cases difficult to identify above plasma radiation 

Different for the narrow ion feature, $S_i(k)$ is large! It reflects the motion of the ions!

**Ion feature**

![Graphs showing ion feature](image)
(a) \( T_i > T_e \)  
Ion acoustic modes are strongly damped, width approx. determined by ion temperature. Increasing \( T_e \) decreases damping, \( \cdots \) 
co- and counter-propagating ion acoustic waves show up, from the shape \( T_i \) and \( T_e \) can be deduced, the electron density \( n_e \) must be obtained from the absolute intensity which requires calibration.

For very large \( \alpha \) and \( T_i < T_e \) the separation of the resonances becomes

\[
(\omega_s - \omega_0)^2 = \frac{Z k_B T_e k^2}{m_i}
\]

\( Z \) is the charge of the ions

Linear function of \( ZT_e \)

\( T_i \) is given by the width of the resonances.
(b) Drift velocities between electron and ions influence damping of the ion acoustic waves asymmetric profiles

Electron Landau damping of wave propagating in the direction of the drift reduces damping increasing peak opposite case for counter-propagating wave

Such measurements are relevant to heat transport studies

(c) Scattering in hydrogen plasma with small amounts of impurities (j)
(d) Impurity peak scales with $n_j \bar{Z}^2$ where $\bar{Z}$ is the mean charge of the impurity ions (j) gives $n_j$ if $\bar{Z}$ is known from spectroscopic observations Width mean thermal speed of impurities
Shift of total ion feature $\rightarrow$ velocity of ion motion in the direction of $\mathbf{k}$

Recent proposal for study of plasma jets in a linear plasma generator

A few examples:
Scattering in a linear pinch discharge with different gas fillings

Peaks correspond to ion acoustic modes

\[
(\omega_s - \omega_o)^2 = \left( \frac{Zk_B T_e}{m_i} \frac{\alpha^2}{1 + \alpha^2} + \frac{3k_B T_i}{m_i} \right) k^2
\]
Impurities!

Figure 3.5. Measured single-shot scattered light spectra for hydrogen plasmas with noble gas additives. Square points are GMA data, solid line is best fit to theory. Parameters from fitting to theory are (a) 1% argon, $T_e = 35$ eV, $n_e = 8 \times 10^{24}$ m$^{-3}$, $Z^* = 7.5$. (b) 0.29% argon, $T_e = 46$ eV, $n_e = 2 \times 10^{24}$ m$^{-3}$, $Z^* = 7.9$. (c) 4.5% xenon, $T_e = 17.5$ eV, $n_e = 2.5 \times 10^{24}$ m$^{-3}$, $Z^* = 3.8$. (d) 4.8% xenon, $T_e = 17$ eV, $n_e = 1 \times 10^{24}$ m$^{-3}$, $Z^* = 2.7$. 
Examples of spectra obtained in the gas-liner pinch (Z-pinch)

(a) $n_e = 3.8 \times 10^{18} \text{ cm}^{-3}$
$T_e = 15 \text{ eV}$
$n_{\text{imp}} = 7.8 \times 10^{16} \text{ cm}^{-3}$
$x_d = 0.054$
$Z_{\text{imp}} = 3.0$

(b) $n_e = 1.9 \times 10^{18} \text{ cm}^{-3}$
$T_e = 10 \text{ eV}$
$n_{\text{imp}} = 4.8 \times 10^{16} \text{ cm}^{-3}$
$x_d = -0.007$
$Z_{\text{imp}} = 3.0$
Laser scattering techniques allow **local** measurements shifting laser beam and line of observation.
Spectrum is observed along scattering vector $\mathbf{k}$

Selection of position in the plasma

(c) information of radial velocities and drifts

(d) azimuthal components
Availability of two-dimensional detectors (CCD cameras) drastically expands the experimental capabilities.

Scattering cylinder in the plasma is imaged onto entrance slit of stigmatic spectrograph → each cross-section of the scattering column gives spectrum in the exit plane.

Example:
Plasma parameters in a gas-puff pinch obtained in one shot at one time.
Lasers

Three characteristics of lasers are made use of
(a) High directionality  →  good focusing
(b) Narrow spectral width  →  needed for ion feature
(c) High power  →  needed for scattered intensity larger than plasma radiation

Limit on power: absorption can lead to undesirable heating

Limit on Frequency: \( \omega_0 > \omega_{pe} \)

critical when scattering on high density fusion plasmas

Powerful Gyrotrons (\( \lambda \sim 1 \text{ mm} \))

Scattering in hot fusion plasmas to reach collective regime, considered for \( \alpha \)-particle diagnostics in ITER
Recent developments

Scattering with x-rays on laser-produced plasmas

Scattering on warm dense matter employing FEL’s